Acknowledging and Incorporating Uncertainty in Model-Based Inferences

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Overview

Acknowledging uncertainty in simulation parameters/models:

• Why?
  – Value of the approach
  – Some examples

• How?
  – Methods / Considerations
  – Useful features of simulation tool
  – Some available tools

• NMSUDs: new tool integrating R and NONMEM®
Uncertainty in Models & Parameters

• CTS employ models and parameter values based on a variety of prior information sources and assumptions
• CTS often involve extrapolations to unobserved conditions

Problem: Substantial uncertainty can exist in the models and parameters used for CTS.

A Solution: Acknowledge the uncertainty by formal incorporation in the simulation process.
Why Include Uncertainty in M&S?

• When uncertainty is not included, simulation results are only valid if the model and parameters are truth.

• Including uncertainty allows for a quantitative evaluation of the current state of knowledge
e.g. How confident are you in the simulation results?

• View simulation outcomes as a probability distribution; conditioned on current knowledge

• Results in Global Sensitivity Analysis of simulation outcome dependence on parameter (model) assumptions
e.g. What’s the impact of model deficiencies?
Example 1: Optimal Design of a Pediatric Trial Using Simulations with Uncertainty

In collaboration with John Mondick, Jeff Barrett
The Children's Hospital of Philadelphia
Objectives of the Simulation Study

• To design a pediatric trial given the practical limitations
  – Sparse sampling
  – Time windows that patients are available for sampling

• To power the study to be able to estimate clearance for children < 1 year with sufficient precision and accuracy
Range of Practical Limitations for Trial Design

- 100-200 patients
- Age: 0-18 years
- Dosing: combination of Drug 1 and Drug 2
- Sparse sampling: three samples no later than 6 hours post-dose; one sample at 24-30 or 48-96 hours post-dose.
Specific Aims

- Select sampling times to characterize the population PK model

- Select number of patients sufficient to estimate the parameters with the desired precision

- Select proportion of patients with AGE < 1 year to sufficiently estimate age effect with the desired precision and accuracy
Models

• Drug 1: Three-compartment model parameterized in terms of CL, Q1, Q2, V1, V2 and V3. Characteristic half-lives: 10 minutes, 2 hours, 2 days

• Drug 2: Two-compartment model parameterized in terms of CL, Q, V1, and V2. Characteristic half-lives: 10 minutes, 2 hours
Initial Study Design

- **n=200**
- **Group 1:** 4 samples at
  - 5 to 15 minutes
  - 0.75 to 1.5 hours
  - 3.5 to 4.5 hours
  - 48 - 96 hours (25% of patients)
- **Group 2:** 4 samples at
  - 15 to 30 minutes
  - 2 to 3 hours
  - 5 to 6 hours
  - 48 - 96 hours (25% of patients)

Initial design guided by D-optimality based on typical individual
Initial Design Results: CL

Drug 1

Drug 2

-40
-30
-20
-10
0
10
20
30
40
50
0
10
20
30
40
50

Bias (%)

CL (L/h)

Bias (%)

CL (L/h)
Initial Design Results: \( V \)

Drug 1

Drug 2
Initial Design Results: Drug 1 Bias

![Graph showing bias for different parameters]

- Parameters: V1, V2, V3, CL, Q2, Q3, OMV1, OMCL
- Bias values range from -100 to 200%
Initial Design Results: Drug 2 Bias
What to do?

• Improve our knowledge about population parameters (reduce uncertainty)

• Improve design to make it robust to the assumptions about the model parameters
Final Study Design

• 24 hour sample added in 50% of patients
• Patients with a sample collected 48 – 96 hours increased to 50%
• Sample fixed at 5 minutes included for both schedules
• Sampling windows adjusted for remaining times
Final Design Results: CL

Drug 1

Drug 2

Bias (%) vs. CL (L/h) for Drug 1 and Drug 2.
Final Design Results: V

Drug 1

Drug 2
Final Design Results: Drug 1 Bias
Final Design Results: Drug 2 Bias

![Graph showing bias for different variables: V1, V2, CL, Q, OMV1, OMCL. The y-axis represents bias in percentage, ranging from -50 to 200, and the x-axis lists the variables.]
Conclusions

• Design was modified to make it robust to uncertainty across parameters

• Given the PK sample timing limitations, PK for both drugs could be accurately assessed

• 200 patients sufficient to characterize PK of both drugs

• 50 patients needed < 1 year old to characterize the suspected age effect on clearance
Example 2: Evaluation of Trial Design and Dose/Regimen Selection

Hypothetical Example
Specific Aims

Select dose
• To maximize % of patients with PD response at trough within a specific interval

Estimate (for a given design/dosing rule)
• % of patients with PD response at trough above and below the specified interval (goal = 90% of patients in target range)
Simulation Model

Study design:
- Oral administration
- Steady-state BID dosing
- 1000 patients

PK model:
- 3-compartment model;
- Terminal half-life ~ 30 hours

PK/PD model:
- direct Emax model
Simulations: Dose Selection Step

• Assuming perfect knowledge of population parameters, simulate study and compute expected endpoint values

• Assuming dose linearity, select the best dose that maximizes % of patients in the desired exposure range
Simulations: Sensitivity Analysis

- Conduct simulations with uncertainty to estimate range of possible outcomes
- Identify the most influential parameters
- Evaluate the effect of extra knowledge (decreased uncertainty)
No Uncertainty in Model Parameters

Effect-time course:

Black: median
Yellow: 80% CI
Red: 90% CI
Green: 95% CI

Dashed: desired range of trough effect (97% of patients were inside of this range)
Uncertainty in PK Parameters

Effect of uncertainty in CL: % of patients with trough effect within the desired range

Simulated CL:
Black: median
Red: 95% CI

Conclusion: Uncertainty in CL is less important than uncertainty in PK/PD model parameters
Uncertainty in PD Parameters

Effect of uncertainty in EMAX: % of patients with trough effect within the desired range

Simulated EMAX:
Black: median
Red: 95% CI

Conclusion: Precise knowledge of EMAX is very important
Uncertainty in PD Parameters

Effect of uncertainty in EC50: % of patients with trough effect within the desired range

Simulated EC50:
Black: median
Red: 95% CI

Conclusion: Uncertainty in EC50 is less important than uncertainty in EMAX
Conclusions for Example 2

• When conditioned on current level of knowledge, this design results in:
  \[ P(>90\% \ of \ patients \ in \ target \ range) = 0.67 \]

• Doses planned for the study are high enough so that exposure or EC50 are not as important as EMAX

• Improved estimates of EMAX may significantly improve precision of the simulation predictions of trial outcomes
Example 3: View Population Variability and Uncertainty in Prediction For New Dose & Regimen

Competing therapy mean response at 2 weeks.
Sensitivity of Simulation Endpoint to Parameter Assumptions
## Example 4: Phase III Trial Simulation

### Results of Local Sensitivity Analysis

<table>
<thead>
<tr>
<th>Fixed Value of ZDVSL</th>
<th>% Trials Successful&lt;sup&gt;a&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>30.6%</td>
</tr>
<tr>
<td>0.5</td>
<td>70.4%</td>
</tr>
<tr>
<td>0.735</td>
<td>93.0%</td>
</tr>
<tr>
<td>1.0</td>
<td>99.0%</td>
</tr>
</tbody>
</table>

<sup>a</sup>Results reflect 500 simulated trials of 2000 patients
Global Sensitivity Analysis:
Probability of Successful Trial vs. Uncertainty
Sensitivity Analysis Surface: Most Influential Parameters
Example 5: Covariate Model for Population PK; Inferences in the Face of Uncertainty

Covariate Effects

Typical CL/F (L/hr) = 6.64

Black Race on CL/F
Asian Race on CL/F
Hispanic Race on CL/F
Other Race on CL/F
Age on CL/F
CRCL (IBW) on CL/F
Weight on CL/F
Weight on V/F
Weight on K=CL/V
Example 6: Mean Response Across Parameter Uncertainty

- Mean Placebo Effect vs. P1
- Mean Effect vs. CL
- Mean Effect vs. V
- Mean Effect vs. AUC50
- Mean Effect vs. Emax

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Summary of Examples

• Acknowledge Uncertainty:
  – Predictions of expected responses are viewed in the context of the uncertainty in the simulation parameters (and/or model)

• Impact on Model-Based Inferences:
  – Sensitivity analysis allows for quantitative description of outcome dependencies on model assumptions
  – This approach leads to an informed application of simulation results in the decision making process
  – Implementation requires specific tools but not more CPU-time intensive than simulations without uncertainty
How?
Simulation Plan

• Conventional CTS (without uncertainty):
  – Select model and model parameters
  – Simulate study 1000 times (with the same population parameters but different realizations of individual parameters)
  – Investigate range of possible outcomes (for fixed values of population parameters)
  – Repeat this process for different values of model parameters to investigate sensitivity of the results to assumptions (requires multiple repeats of simulations)
Simulation Plan

• CTS with uncertainty:
  - Select model and probability distribution of model / parameters (representing uncertainty)
  - Simulate study 1000 times (each time with different values of population parameters drawn from parameter distributions)
  - Investigate range of possible outcomes (given level of current knowledge)
  - Investigate sensitivity of the results to assumptions (does not require additional simulations)
Hierarchy of Random Variability & Uncertainty in Simulation

• Intra-individual, residual error ($\varepsilon$)
  - 1 draw from $(0, \sigma^2)$ per observation, constant fixed-effect parameters ($\theta$)

• Inter-individual error ($\eta$) in parameter
  - 1 draw from $(0, \omega^2)$ per individual, constant fixed-effect parameters ($\theta$)

• Uncertainty in models and parameters
  - 1 draw from prior distribution for $\theta$, $\Omega$, $\Sigma$ per trial
\[ C_t = \frac{D}{V} e^{-\frac{CL_i}{V}t} + \epsilon_t \]
Interindividual Variability: CL

\[ \eta \sim N(0, \omega^2) \]

Residual Variability

\[ \varepsilon \sim N(0, \sigma^2) \]

\[ C_t = \frac{D}{V} e^{-CLi/V^*t} + \varepsilon_t \]
Interindividual Variability: CL

\[ \eta \sim N(0, \omega^2) \]

Residual Variability

\[ \epsilon \sim N(0, \sigma^2) \]

\[ C_t = \frac{D}{V} e^{-CL_i/V \cdot t} + \epsilon_t \]
Simulations with Uncertainty
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\[
C_t = \frac{D}{V} e^{-CLi/V} t + \varepsilon_t
\]

Interindividual Variability: CL
\[\eta \sim N(0, \omega^2)\]

Population Variance in CL

Residual Variability
\[\varepsilon \sim N(0, \sigma^2)\]
$C_t = \frac{D}{V} e^{-\frac{C_L i}{V} t} + \varepsilon_t$

$\theta \sim N(0, \psi^2)$

$\eta \sim N(0, \omega^2)$

$\Omega \sim W^{-1}(\text{mode, df})$

$\varepsilon \sim N(0, \sigma^2)$

$TVCL = \exp(\theta)$

Uncertainty in $\ln(TVCL)$

Uncertainty in Var CL

Interindividual Variability: CL

Residual Variability
Uncertainty in the Model (Competing Models)

- Through Uncertainty in Parameters:
  - Combinations of some parameters approximate a different model structure.
  - e.g. High EC50 approximates linear model

- Simulate from Expected Probability of Each Model:
  
  \[ P(\text{Model A}) = 0.7 \quad \text{P(Model B) = 0.3} \]

  - Draw random uniform variable (0-1), R
  - Model A if \( R \leq 0.7 \); Model B if \( R > 0.7 \)
Obtaining Measures of Uncertainty

• Results from prior modeling exercise
  – Variance-covariance matrix of estimates
  – Bootstrap parameter distributions
  – Bayesian posterior distributions

• Review of literature for ranges of plausible values

• Poll experts (everyone’s view can be part of the simulation)
Assembling Simulation Model Components

Trial Simulation Model

Pop PK Model
(NONMEM; NPBootstrap)

PD Models
Efficacy/ AEs
(BUGS; Posterior Distributions)

Clinical Outcome Model
(Meta-Analysis of Literature Data; Var-Cov Matrix of Estimates)

Trial Conduct Model
(Dropout Rate; Range of Expert Opinions)
Simulation Tool: Requirements

1. Monte Carlo simulation hierarchy with multiple levels of nested random effects (at least 3)
2. Ability to incorporate joint uncertainty distributions from other methods (e.g. bootstrap, Bayesian)
3. Simulation and estimation (ML) for typical population PK and PD systems in same tool
4. Programmable/extensible language with data manipulation and graphics capability
5. Platform neutral (Win, Unix, Linux, Mac OS X)
Current Simulation Tools

Some programs with Monte Carlo simulation capabilities at parameter uncertainty level are available, but not all requirements are met:

- WinBugs
- NONMEM PRI OR subroutine
- Trial Simulator
- Others…
NMSUDs R/ NONMEM Package

1. Generates draws from the uncertainty distributions at inter-trial level, maintaining joint distribution (covariance) of parameters

   -OR-

1. Samples from previously determined uncertainty distributions (e.g. Bootstrap, Bayesian Posteriors)

2. Generates NONMEM control streams for simulation (estimation)

3. Runs NONMEM or R for simulation (and possibly estimation) of each trial

4. Summarizes the results of each trial and across all trials
<table>
<thead>
<tr>
<th>Parameter type (NONMEM name)</th>
<th>Distribution</th>
<th>Parameters of the distribution</th>
<th>Implementation</th>
<th>How to assign distribution parameters based on NONMEM run</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single uncorrelated population parameter (THETA)</td>
<td>Normal</td>
<td>Mean $\mu$, variance $\sigma^2$</td>
<td>Standard R function <code>rnorm(., \mu, \sigma)</code></td>
<td>$\mu$: population parameter estimate; $\sigma$: standard error of the parameter estimate.</td>
</tr>
<tr>
<td>Set of correlated population parameters (THETA)</td>
<td>Multivariat Normal</td>
<td>Vector of mean values $\mathbf{M}$, variance-covariance matrix $\mathbf{\Sigma}$</td>
<td>Standard R function <code>mvnorm(., \mathbf{M}, \mathbf{\Sigma})</code></td>
<td>$\mathbf{M}$: vector of population parameter estimate; $\mathbf{\Sigma}$: variance-covariance matrix of the parameter estimates.</td>
</tr>
<tr>
<td>Variance of the random effect (OMEGA)</td>
<td>Scaled Inverse $\chi^2$</td>
<td>Number of degrees of freedom $\nu$, scale $s^2$.</td>
<td>Standard R function <code>rchisq(., \nu)</code></td>
<td>$\nu$: number of patients used to obtain the estimate; $s^2$: estimated variance of the random effect.</td>
</tr>
<tr>
<td>Variance-covariance matrix of the random effects (OMEGA)</td>
<td>Inverse Wishart</td>
<td>Number of degrees of freedom $\nu$, scale matrix $\mathbf{S}$. Implicit parameter is the $\mathbf{S}$ matrix dimension $k$.</td>
<td>Proprietary R function <code>myriwish(k, \nu, \nu \mathbf{S})</code> based on the standard <code>riwish()</code> function</td>
<td>$\nu$: number of patients used to obtain the estimate; $\nu \mathbf{S}$: estimated variance-covariance matrix of the random effect.</td>
</tr>
<tr>
<td>Variance of the error term (SIGMA)</td>
<td>Scaled Inverse $\chi^2$</td>
<td>Number of degrees of freedom $\nu$, scale $s^2$.</td>
<td>Standard R function <code>rchisq(., \nu)</code></td>
<td>$\nu$: number between the number of patients and the number of observations used to obtain the estimate; $s^2$: estimated variance of the error.</td>
</tr>
</tbody>
</table>
Sample from Uncertainty Distributions

```r
SourceDirName <- "c:/code/NMSUDSalpha1/Scripts/"
DirName <- "c:/code/NMSUDSalpha1/Example2/"
FigureDir <- DirName
source(paste(SourceDirName,"SimulationFromFileJan30.R",sep=""))
source(paste(SourceDirName,"CreateParametersOct24.R",sep=""))
source(paste(SourceDirName,"PostProcessingOct20.R",sep=""))

ThetaMean <-c(100,1000,7500,500,0.5,1,500)
ThetaCovar <- diag(c(150,15000,1000000,6400,0.1,0.04,15000))
OmegaModeList <-list(0.04,0.09,1,0.09,0.25)
OmegaDfList <-c(50,50,50,50)
SigmaModeList <- list(0.04,0.04)
SigmaDfList <- c(75,75)

# this part ensures reproducability:
set.seed(123)
rrnorm(5)
# -0.56047565 -0.23017749 1.55870831 0.07050839 0.12928774

NsimPar <- 14
nsim <- 100
parameters <- CreateParametersForSimulation(nsim=1.5*nsim,
                           ThetaMean=ThetaMean,ThetaCovar=ThetaCovar,
                           OmegaModeList=OmegaModeList,OmegaDfList=OmegaDfList,
                           SigmaModeList=SigmaModeList,SigmaDfList=SigmaDfList)

bounds <- data.frame(par =1:NsimPar,lower =rep(0,NsimPar),upper=rep(Inf,NsimPar))
parametersTruncated <- TruncateParametersForSimulation(parameters, bounds)
parametersTruncated <- parametersTruncated[1:nsim,]
write.table(parametersTruncated, file=paste(DirName,"Example2Par.csv",sep=""),
            quote = F,sep="",row.names = F,col.names = F)
```
Sample from Uncertainty Distributions

```r
SourceDirName <- "c:/code/NMSUDSAlpha1/Scripts/
DirName  <- "c:/code/NMSUDSAlpha1/Example1/
source(paste(SourceDirName,"SimulationFromFileApr2006.R",sep=""))
source(paste(SourceDirName,"CreateParametersApr2006.R",sep=""))
source(paste(SourceDirName,"PostProcessingOct28.R",sep=""))

ThetaMean  <- c(11.8,85)
ThetaCovar <- matrix(c(0.232,0.449,0.449,12.8),2,2)
OmegaModeList <- matrix(c(0.0572,0.011,0.011,0.0615),2,2)
OmegaDfList <- 20
SigmaModeList <- 0.0454
SigmaDfList <- 200

# this part ensures reproducability:
set.seed(123)
rnorm(5)
# -0.56047565 -0.23017749  1.55870831  0.07050839  0.12928774

parameters <- CreateParametersForSimulation(nsim=100,
    ThetaMean=ThetaMean,ThetaCovar=ThetaCovar,
    OmegaModeList=OmegaModeList,OmegaDfList=OmegaDfList,
    SigmaModeList=SigmaModeList,SigmaDfList=SigmaDfList)

bounds <- data.frame(par = c(1,2),lower = c(5,30),upper= c(20,150))
parametersTruncated <- TruncateParametersForSimulation(parameters, bounds)
write.table(parametersTruncated, file=paste(DirName,"Example1Par.csv",sep=""),
    quote = F,sep="",row.names = F,col.names = F)
```
Parameters Generated from Uncertainty Distributions
(or Bootstrap, Bayesian Posteriors, etc.)

1 full set of simulation parameters per trial
(each row = 1 trial)
Typical NONMEM $SIM Control Stream

$PROB RUN# 001
$INPUT C ID AMT TIME EVID DV WT SS II
$DATA ../Example2Data.csv IGNORE=@
$SUBROUTINE ADVAN4 TRAN4
$PK
   TVCL = THETA(1)*(WT/70)**0.75
   TVV2 = THETA(2)*WT/70
   V3  = THETA(3)*WT/70
   Q   = THETA(4)*(WT/70)**0.75
   CL  = TVCL*EXP(ETA(1))
   V2  = TVV2*EXP(ETA(2))
   F1  = 2
   S2  = V2/1000
   T1  = TVCL/TVV2
   T23 = Q/TVV2
   T32 = Q/V3
   TL1 = (T1+T23+T32)+SQRT((T1+T23+T32)**2-4*T1*T32))/2
   TVKA = THETA(5)+TL1
   KA  = TVKA*EXP(ETA(3))
   EMAX = THETA(6)*EXP(ETA(4))
   EC50 = THETA(7)*EXP(ETA(5))
$ERROR
   CONC=A(2)/S2
   EFF = EMAX*CONC/(EC50+CONC)
   Y=EFF*EXP(EPS(1))
   IPRED=CONC*EXP(EPS(2))

$THETA
   100   ; 1 TVCL
   1000  ; 2 TVV2
   7500  ; 3 TVV3
   500   ; 4 TVQ
   0.5   ; 5 TVKA
   1     ; 6 EMAX
   500   ; 7 EC50
$OMEGA
   0.04  ; 1 CL
   0.09  ; 2 V2
   1.00  ; 3 KA
   0.09  ; 4 EMAX
   0.25  ; 5 EC50
$SIGMA
   0.01  ; 1 EFF
   0.04  ; 2 PK
$SIMULATION (12345) (6789 UNIFORM)
$TABLE EVID TIME CONC IPRED EFF DV NOPRINT NOHEADER
   NOAPPEND FILE=../001.tab
Constraining Simulated Parameters

• When simulating from a multi-variate Normal covariance matrix, use caution about plausible values for population-level parameters.
• Constrain model so that plausible values are simulated, e.g.:
  LNCL=THETA
  CL=EXP(LNCL)
• Bootstrap distributions and Bayesian posteriors may already be constrained to plausible values
Simulate from Uncertainty Distributions using NONMEM Model Control Stream

```r
SimulateFromFile(DirName, CtlFileName="Example2.ctl",
Parameters=parametersTruncated,
OutputFileName="Example2Output.csv",
 ntheta=7, nomegaList=c(1,1,1,1,1), nsigmaList=c(1,1),
 NONMEMcommand="perl rmbatch.p c:/code/NMSUDSAlpha1/Example2/ sim",
SimTabFileName="c:/code/NMSUDSAlpha1/Example2/sim.tab",
 CreateSummary=T, SummaryFunction=MySummaryFunction)
```
NMSUDs R/ NONMEM Package

Open-source tool, distributed under GPL.

Download alpha version of code from:
www.metruminstitute.org/downloads

Forward questions/comments to:
NMSUDs@metruminstitute.org
Downloads

The following are available for download from this site.

- **NMQual resources**: Download source code and documentation for Metrum Institute's free NONMEM® installer.

- **Qpharm Update**: A periodical of quantitative pharmacology, including reviews and summaries of methods and applied research, discussion groups, recent meetings and upcoming events.

- **G77 utility**: Download Metrum Institute's redistribution of a popular Fortran77 compiler for Windows. Works great with NMQual!

- **MD5 utility**: Don't have your own MD5 checksum software? Here's a simple Perl tool that can help you verify other downloads on this page.

- **NMSUDS**: Download source code and documentation for Metrum Institute's NMSUDS (alpha 1 release) toolbox. This R/ NONMEM® Toolbox for Simulations from Uncertainty DistributionS allows the implementation of parameter uncertainty as an additional level in the random effects hierarchy and can be used with simulation models defined in NONMEM® and/or R. As the version indicates, this is work in progress; feel free to experiment, and use at your own risk. Please provide feedback to NMSUDS@metruminstitute.org. NMSUDS is distributed under GPL.

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• Industry collaborators (examples)
Questions or Comments?